ELEMENTARY LAWS AND RESULTS ON RATION AND PROPORTION

1. $\frac{a}{b}>1$ Implies $\frac{a}{b}>\frac{a+x}{b+x}$ $(x>0)$

2. $\frac{a}{b}<1$ implies $\frac{a}{b}<\frac{a+x}{b+x}$ $(x>0)$

3. If $\frac{a_1}{b_1},\frac{a_2}{b_2},....,\frac{a_n}{b_n}$ are unequal fractions, of which the denominators are of the same sign, then the fraction $\frac{a_1+a_2+...+a_n}{b_1+b_2+...+b_n}$ lies, in magnitude, between the greatest and least of them.

4. If $\frac{a_1}{b_1}=\frac{a_2}{b_2}=...=\frac{a_k}{b_k}$, then each of these ratios is equal to $\left(\frac{\alpha_1 a_1^n+\alpha_2 a_2^n+...+\alpha_k a_k^n}{\alpha_1 b_1^n+\alpha_2 b_2^n+...+\alpha_k b_k^n}\right)^{1/n}$, where n is an integer.

5. $a,\ b,\ c,\ d$ are in proportion. Then

(i) $\frac{a}{b}=\frac{c}{d}$

(ii) $\frac{a}{c}=\frac{b}{d}$ (alternendo)

(iii) $\frac{a+b}{b}=\frac{c+d}{d}$ (componendo)

(iv) $\frac{a-b}{b}=\frac{c-d}{d}$ (dividendo)

(v) $\frac{a+b}{a-b}=\frac{c+d}{c-d}$ (componendo and dividendo)

(vi) $\frac{b}{a}=\frac{d}{c}$ (invertendo)